TWO MACHINE FLOW SHOP SCHEDULING PROBLEM WITH UNLIMITED INTERMEDIATE STORAGE

F2||Cmax Problem

Consider a flow shop with two machines in series with unlimited storage between the two machines. There are n jobs, the processing time of job j on machine 1 is p1j, and the processing time on machine 2 is p2j. This was one of the first problems to be analyzed in the early days of Operations Research and led to a classical paper in scheduling theory by S.M. Johnson. The rule that minimizes the makespan is commonly referred to as Johnson's rule.

SPT(1)-LPT(2) Schedule

An optimal sequence for F2||Cmax problem can be described as follows:

- Step 0:
  Partition the jobs into two sets.
  $O = \{j: p1j < p2j\}, j=1\ldots n$
  $R = \{j: p1j > p2j\}, j=1\ldots n$
  The jobs with p1j=p2j may be put in either set.
- Step 1:
  Put the jobs in O into schedule S in increasing order of p1j (SPT).
  Ties may be broken arbitrarily.
  $S = (\ldots i, k, l, \ldots)$ where $p1i < p1k < p1l$, $i,k,l \in O$
- Step 2: Add the jobs in R into schedule S in decreasing order of p2j (LPT).
  Ties may be broken arbitrarily.
  $S = (\ldots, i, k, l, \ldots, t,u,v, \ldots)$ where $p2t > p2u > p2v$, $t,u,v \in R$

A schedule S is referred to as a SPT(1)-LPT(2) schedule. Of course there may be more than one schedule constructed this way.

Case

There are 10 jobs to be scheduled. The processing times on two machines:

Table 1: Processing times of jobs on two machines

Job ID	p_{1j}	p_{2j}
1	6	3
2	4	8
3	12	5
4	6	11
5	10	6
6	6	4
7	3	10
8	15	7
9	9	11
10	10	2

In the sheet “Jobs”, there should be a button. When one clicks the button, the Optimal Sequence table should be filled.

	Mach. 1	Mach. 2	Optimal Sequence
J1	6	3	
J2	4	8	
J3	12	5	
J4	6	11	
J5	10	6	
J6	6	4	
J7	3	10	
J8	15	7	
J9	9	11	
J10	10	2	